Second quantized "fermions" with integer spins

Understanding Nature with the Spin-Charge-Family theory

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Some publications:

- *Phys. Lett.* **B 292**, 25-29 (**1992**), *J. Math. Phys.* **34**, 3731-3745 (**1993**), *Mod. Phys. Lett.* **A 10**, 587-595 (**1995**), *Int. J. Theor. Phys.* **40**, 315-337 (**2001**),
- *Phys. Rev.* **D 62** (04010-14) (**2000**), *Phys. Lett.* **B 633** (**2006**) 771-775, **B 644** (**2007**) 198-202, **B** (**2008**) 110.1016, *JHEP* **04** (**2014**) 165, *Fortschritte Der Physik-Progress in Physics*, **(2017) with** H.B.Nielsen,
- *Phys. Rev.* **D 74** 073013-16 (**2006**), **with** A. Borštnik Bračič,
- *New J. of Phys.* **10** (**2008**) 093002, arxiv:1412.5866, **with** G.Bregar, M.Breskvar, D.Lukman,
- *Phys. Rev.* **D** (**2009**) 80.083534, **with** G. Bregar,
- *New J. of Phys.* (**2011**) 103027, *J. Phys. A: Math. Theor.* **45** (**2012**) 465401, *J. Phys. A: Math. Theor.* **45** (**2012**) 465401, *J. of Mod. Phys.* **4** (**2013**) 823-847, arxiv:1409.4981, **6** (**2015**) 2244-2247, *Phys. Rev.* **D 91** (**2015**) 6, 065004, . *J. Phys.: Conf. Ser. 845 01* **IARD 2017**, *Eur. Phys. J.C.* **77** (2017) 231, **[arXiv:1082.05554v4]**

Second quantized "fermions" with integer and half integer spins

- **Algebras in Clifford space in $d \geq (13+1)$, if used to describe the internal degrees of freedom of fermions — spins and charges of quarks and leptons and antiquarks and antileptons – offer the anticommutation relations among creation and annihilations operators for fermions without postulating the anticommutation relations. Correspondingly these algebras explain the Dirac postulates for the second quantized fermions.**

- **Algebra in Grassmann space offers the second quantized "fermions" with integer spins and in $d \geq 5$ the charges in adjoint representations, fulfilling the anticommutation relations without postulating them.**

- **In d-dimensional Grassmann space of anticommuting coordinates θ^a's**,
  $\{\theta^a, \theta^b\}_+ = 0$, $a = (0, 1, 2, 3, 5, \ldots.., d)$,
  there are 2^d operators ("vectors"), which are superposition of products of θ^a.
- **There are as well derivatives with respect to θ^a's**,
  $\frac{\partial}{\partial \theta_a}$**'s**,
  which are Hermitian conjugated to θ^a's,
  $(\theta^a)^\dagger = \eta^{aa} \frac{\partial}{\partial \theta_a}$,
  $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$,
  which again form 2^d operators ("vectors").
- **Grassmann space offers correspondingly $2 \cdot 2^d$ degrees of freedom.**

- $$\{\theta^{a}, \theta^{b}\}_{+} = 0\,, \qquad \{\frac{\partial}{\partial\theta_{a}}, \frac{\partial}{\partial\theta_{b}}\}_{+} = 0\,,$$
$$\{\theta_{a}, \frac{\partial}{\partial\theta_{b}}\}_{+} = \delta_{ab}\,,$$
$$(\theta^{a})^{\dagger} = \eta^{aa}\frac{\partial}{\partial\theta_{a}}\,,$$
$$(\frac{\partial}{\partial\theta_{a}})^{\dagger} = \eta^{aa}\theta^{a}\,,$$
$$(a, b) = (0, 1, 2, 3, 5, \cdots, d)\,.$$

- **One can arrange products of θ^{a} into 2^{d} irreducible representations with respect to the Lorentz group with the generators**

$$S^{ab} = i\,(\theta^{a}\frac{\partial}{\partial\theta_{b}} - \theta^{b}\frac{\partial}{\partial\theta_{a}})\,.$$

- It is useful to make a choice of the Cartan subalgebra of the Lorentz algebra

$$\mathbf{S}^{03}, \mathbf{S}^{12}, \mathbf{S}^{56}, \cdots, \mathbf{S}^{d-1\;d}\,, \tag{1}$$

- and choose the irreducible representations of the Lorentz group to be the "eigenvectors" of the Cartan subalgebra.

$$\mathbf{S}^{ab}\,\frac{1}{\sqrt{2}}\,(\theta^a + \frac{\eta^{aa}}{ik}\theta^b) = k\,\frac{1}{\sqrt{2}}(\theta^a + \frac{\eta^{aa}}{ik}\theta^b)\,,$$

$$\mathbf{S}^{ab}\,\frac{1}{\sqrt{2}}\,(1 + \frac{i}{k}\theta^a\theta^b) = 0\,.$$

$k = \pm i\,,$ either $a = 0$ or $b = 0$ $\quad k = \pm 1$, otherwise.

- ***k* represents an integer spin**,
$\mathbf{S}^{03}\;\frac{1}{\sqrt{2}}\,(\theta^0 \mp \theta^3)$ **has** $k = \pm i$**, all the others have** $k = \pm 1$.

- The Hermitian conjugated representations of (odd and even) products of θ^a are

$$\begin{aligned}
\frac{1}{\sqrt{2}}(\theta^a + \frac{\eta^{aa}}{ik}\theta^b)^\dagger &= \eta^{aa}\,\frac{1}{\sqrt{2}}(\frac{\partial}{\partial\theta_a} + \frac{\eta^{aa}}{i(-k)}\,\frac{\partial}{\partial\theta_b})\,,\\
\frac{1}{\sqrt{2}}\,(1 + \frac{i}{k}\,\theta^a\theta^b)^\dagger &= \frac{1}{\sqrt{2}}\,(1 + \frac{i}{k}\,\frac{\partial}{\partial\theta_a}\,\frac{\partial}{\partial\theta_b})\,.
\end{aligned}$$

- Let us notice that
$\frac{\partial}{\partial\theta_a}$ **on a "state" which is just an identity, $\mid I >$, gives zero**,
$\frac{\partial}{\partial\theta_a} \mid I > = 0$,
while $\theta^a \mid I >$, **or any superposition of products of θ^a's applied on $\mid I >$, gives the "vector" back.**

- **Let us make a choice of the vacuum state in the Grassmann case as $|1>$.**
- Let us recognize again that

$$\{\theta^{\mathbf{a}}, \theta^{\mathbf{b}}\}_{+} = \mathbf{0}\,, \quad \{\frac{\partial}{\partial\theta_{\mathbf{a}}}, \frac{\partial}{\partial\theta_{\mathbf{b}}}\}_{+} = \mathbf{0}\,,$$

$$\{\theta_{\mathbf{a}}, \frac{\partial}{\partial\theta_{\mathbf{b}}}\}_{+} = \delta_{\mathbf{ab}}\,,$$

$$(\theta^{\mathbf{a}})^{\dagger} = \eta^{\mathbf{aa}}\frac{\partial}{\partial\theta_{\mathbf{a}}}\,,$$

$$(\frac{\partial}{\partial\theta_{\mathbf{a}}})^{\dagger} = \eta^{\mathbf{aa}}\theta^{\mathbf{a}}\,,$$

Let us recognize that:
θ^{a}'s have all the properties of creation operators ,
$\frac{\partial}{\partial\theta_b}$'s have all the properties of annihilation operators ,
provided that **Identity does not contribute to the representations** ,
since $I|1>= I|1>$
and
$I^{\dagger}\;|I>=|1>$, **it is NOT zero** .

- **Let us use in $d=2(2n+1)$, n is a positive integer, for the starting Grassmann odd "vector" the notation**

$$\hat{b}^{\theta 1\dagger}_{1} := (\frac{1}{\sqrt{2}})^{\frac{d}{2}} (\theta^0 \pm \theta^3)(\theta^1 + i\theta^2)(\theta^5 + i\theta^6)\cdots(\theta^{d-1} + i\theta^d)\,,$$
$$(\hat{b}^{\theta 1\dagger}_{1})^{\dagger} = \hat{b}^{\theta 1}_{1} = (\frac{1}{\sqrt{2}})^{\frac{d}{2}} (\frac{\partial}{\partial\theta^{d-1}} - i\frac{\partial}{\partial\theta^{d}})\cdots(\frac{\partial}{\partial\theta^{0}} - \frac{\partial}{\partial\theta^{3}})\,.$$

  $\hat{b}^{\theta 1}_{1}$ is the Hermitian conjugate $(\hat{b}^{\theta 1\dagger}_{1})^{\dagger}$.
- **All the rest creation operators follow from the starting one by the application of $\mathbf{S}^{ab}$.**
- **All the annihilation operators follow from the creation operators by the Hermitian conjugation.**

- Let those "vectors" belonging to different irreducible representations be denoted by **since** $\hat{b}_j^{\theta k\dagger}$ and their Hermitian conjugated partners by **since** $\hat{b}_j^{\theta k} = (\hat{b}_j^{\theta k\dagger})^\dagger$.
  Then it follows

$$
\begin{aligned}
\{\hat{b}_i^{\theta k}, \hat{b}_j^{\theta l\dagger}\}_+ | 1 > &= \delta_{ij}\, \delta^{kl} \,| 1 > , \\
\{\hat{b}_i^{\theta k}, \hat{b}_j^{\theta l}\}_+ | 1 > &= 0 \,| 1 > , \\
\{\hat{b}_i^{\theta k\dagger}, \hat{b}_j^{\theta l\dagger}\}_+ \,| 1 > &= 0 \,| 1 > , \\
\hat{b}_j^{\theta k} \,| 1 > &= 0 \,| 1 > \ . \\
\hat{b}_i^{\theta k\dagger} \,| 1 > &= |\phi^k_{i\mathrm{b}} > \ .
\end{aligned}
\qquad (2)
$$

- **These anticommutation relations are just the relations among creation and annihilation operators required by Dirac for fermions.**
  Fermion states correspondingly follow by the application off creation operators on the vacuum state $| 1 >$.

- **Grassmann "fermions" have INTEGER spin!!!**
- Let me suggest the action for the Grassmann "fermions", leading to equations of motion for either odd or even Grassmann "vectors".

$$\mathcal{A}_G = \int d^dx \, d^d\theta \, \omega \, \{\phi^{\dagger}(1 - 2\theta^0 \frac{\partial}{\partial \theta^0}) \, \frac{1}{2} \, \theta^a p_a \phi\} + h.c. \, .$$

- **Could fermions with integer spin exist as elementary fields?**